Response to Reviewer #2

We thank the reviewer for the constructive criticism and suggestions. We have followed your suggestions and revised the manuscript accordingly. Please find our responses below.

The paper addresses the depth distribution of particulate organic matter (POM) and its associated flux, introducing analytical solutions and a numerical approach to solve corresponding differential equations. The equations of the analytical part are based on the assumption that mass and sinking speed of a particle is governed by its diameter, which varies with time due to the degradation processes. Time-varying solutions of differential equation for POM concentration and flux are given for constant and time-varying degradation rates and are finally converted into corresponding depth-varying solutions.

As a reader who does not juggle with DGL solutions every day, I find the derivation of the formulas difficult to track. Readers should be able to do so without the need of many calculations on an extra sheet of paper. E.g., you might want to write the integrals that convert sinking speed formulas (9)/(22) into the corresponding depth formulas (10)/(23).

Answer. Thank you for the suggestion. We extended the text and changed formulae accordingly for AID model:

Combining Eq. (7) and Eq. (3) yields the change in the particle diameter over time as

$$d = d_0 \exp\left(-\frac{\gamma_0 t}{\zeta}\right). \tag{8}$$

Assuming the quasiequilibrium descent of the particle in the Stokes regime, as described by Eq. (4), and taking into account that $W_{p,d} = \partial z'/\partial t$, we estimate the dependence of the particle depth z' on t using Eq. (8):

$$\frac{\partial z'}{\partial t} = c_w d_0^\eta \exp\left(-\frac{\eta\gamma_0 t}{\zeta}\right). \tag{9}$$

By integrating Eq. (9) from the initial particle depth $z' = 0$ at $t = 0$, we find the vertical path travelled by the particle:

$$z' = \frac{\zeta c_w d_0^\eta}{\eta\gamma_0}\left[1 - \exp\left(-\frac{\eta\gamma_0 t}{\zeta}\right)\right]. \tag{10}$$

By eliminating time from Eqs. (9) and (8) by using Eq. (10), we obtain $W_{p,d}$ and d as functions of z':

$$W_{p,d} = H(z')c_w d_0^\eta (1 - \psi z'), \tag{11}$$

$$d = H(z')d_0 (1 - \psi z')^{\frac{1}{\eta}}, \tag{12}$$

and for ADD model:

where α [d] and β are empirical constants. We define such a model as an age-dependent degradation rate (ADD) model. The time dependencies of d and $W_{p,d} = \partial z'/\partial t$ with parameterization of the degradation rate Eq. (20) are obtained similarly to those in Section 3.1. They are expressed as

$$d = d_0 \left(\frac{\alpha}{\alpha + t}\right)^{\beta/\zeta}, \tag{21}$$

$$\frac{\partial z'}{\partial t} = c_w d^\eta \left(\frac{\alpha}{\alpha + t}\right)^{\eta\beta/\zeta}. \tag{22}$$

Integrating Eq. (22) from the initial particle depth $z' = 0$ at $t = 0$, we find the path travelled by a sinking particle as

$$z' = c_w d_0^\eta \frac{\alpha\zeta}{\zeta - \eta\beta} \left[\left(1 + \frac{t}{\alpha}\right)^{(\zeta - \eta\beta)/\zeta} - 1\right]. \tag{23}$$

By eliminating time from Eqs. (20), (21) and (22), we obtain depth-dependent solutions in the same way as in Eqs. (11)-(12):

$$W_{p,d}(z') = c_w d_0^\eta \left(1 + \phi z'\right)^{-\frac{\eta\beta}{\zeta - \eta\beta}}, \tag{24}$$

$$\gamma(z') = \frac{\beta}{\alpha}\left(1 + \phi z'\right)^{-\frac{\zeta}{\zeta - \eta\beta}}, \tag{25}$$

$$d(z') = d_0 \left(1 + \phi z'\right)^{-\frac{\beta}{\zeta - \eta\beta}}, \tag{26}$$

The derivation of $C_{p,d}(0)$ was also extended:

The solution (14) describes the vertical profile of the POM concentration for particles of diameter d under the prescribed particle size distribution $N(d_0)$ [m^{-4}] at $z' = 0$. This distribution can be approximated by the power dependence (e.g., Kostadinov et al., 2009)

$$N(d_0) = M_0 d_0^{-\epsilon},$$

where ϵ is a power-law exponent and M_0 is a constant that can be estimated from the total concentration of sinking POM at $z' = 0$. To obtain the size distribution of $C_{p,d}(0)$, we use a small increment of particle size Δd_0 under the assumption that the concentration is uniform within the interval Δd_0. Then, the distribution $C_{p,d}(0)$ as a product of $N(d_0)$, $m_{0,d}$ and Δd_0 is given by

$$C_{p,d}(0) = M_0 d_0^{-\epsilon} m_{0,d} \Delta d_0 = M_0 c_m d_0^{\zeta - \epsilon} \Delta d_0. \tag{15}$$

I also would like to see some details about how the depth-varying solutions for sinking speed, particle diameter [and degradation rate] and finally the depth-dependent particle concentration and flux are calculated, i.e., how does (10) applied to (8) and (9) yield (11) and (12) (and finally (14)), and how does (23) applied to (20)-(22) yield (24)-(26) (and finally (28)).

Answer. See the answer to the previous comment.

Also, I do not see if formulas (31) (for constant degradation) and (32) (for time-varying degradation), which consider the special case of a constant particle sinking speed, are derived from formulas (14) and (28), respectively. This would be good to see, because (31) and (32) are used to discuss the differences of solutions with respect to the corresponding assumptions.

Answer. Thank you for pointing out this issue. We refined the text in L. 162 accordingly:

The obtained analytical solutions have several important properties. First, we compare these solutions with the solutions obtained under the assumption of a constant sinking velocity when

$$W_{p,d} = c_w d_0^\eta.$$

The solution of Eq. (1) for a constant degradation rate γ corresponds to the exponential profile of the particle concentration

$$C_p(z', d_0) = C_p(0, d_0) \exp\left(-\frac{\gamma_0 z'}{c_w d_0^\eta}\right), \tag{31}$$

whereas the time-dependent degradation rate (20) corresponds to the power-law distribution of the POM concentration

$$C_p(z', d_0) = C_p(0, d_0) \left(\frac{\alpha c_w d_0^\eta}{\alpha c_w d_0^\eta + z'}\right)^\beta. \tag{32}$$

Both of these solutions are frequently used to approximate observed particle flux profiles, e.g., (Martin et al., 1987; Lutz et al., 2002). Notably, a solution of the form (32) can alternatively be obtained under the assumption of a constant degradation rate and a linear increase in the sinking velocity (Kriest and Oshlies, 2008; Cael and Bisson , 2018).

The paper also provides a numerical solution algorithm for particle concentration and flux. The Algorithm is verified w.r.t. the derived analytical solutions (Fig. 1) and applied under the additional assumptions that particle flux is also influenced by (i) temperature and (ii) oxygen concentration. Results indicate a clear dependence on the temperature profile and on parameters with uncertain range, e.g., the exponent mu which relates particle diameter and sinking speed.

In the corresponding sections 4 and 5 I found some places where S_p was used instead of C_p to refer to POM concentration (Algorithm 3, Figure 5, line 269), please correct.

Answer. Thank you. We corrected the text and figures accordingly.

You may want to place the legend of figures 4-6 (which repeats in every single panel plot) only once to the right of the panels.

Answer. Thank you for the suggestion. We changed the legends accordingly.

I would add the explanation that the three columns of panels in figures 4-6 correspond to the model without dependency of temperature and oxygen (panels a and d), additional temperature dependence (panels b and e), and both additional dependencies (panels c and f) to the figure captions in order to make the figures self-explaining.

Answer. Thank you for the suggestion. We have added the proposed text to the figure captions.

Figs 4, 5 and 6 “Three columns of panels correspond to the model without dependency of temperature and oxygen (panels a and d), additional temperature dependence (panels b and e), and both additional dependencies (panels c and f).”

In line 280, I interpreted "1/r < 1" and "r > 1" as defining assumptions at first glance (which makes actually no sense without the definition of r). I would define r=1.25 first and then derive p_min and p_max by the ratios 1/r and r, respectively (probably without extra stressing "1/r<1" and "r>1").

Answer. Thank you for the suggestion. We reworked the text in the Supplement accordingly.

P S2 L4 “The range for parameters is defined for a constant ratio $r>1$. The minimum parameter value p_{min} was set to be proportional to the reference value with a ratio value $1/r$, while the maximum value p_{max} was set to be proportional to the reference value p_{ref} with a ratio value r. For parameters in Table S2, the value of r was chosen to be the same (r=1.25), which satisfies the ranges of all parameters.”